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Generational differences in University Students: Challenges or opportunities?

ARTICLES

Understanding of Algebra of secondary school mathematics teachers from different generations: A qualitative comparative analysis of the answers

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Understanding of Algebra of secondary school mathematics teachers from different generations: A qualitative comparative analysis of the answers

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Abstract: When the challenges faced in the transition period from arithmetic to algebra are considered, it is important to research the differences between secondary school mathematics teachers from Generations X and Y, and their awareness levels. This study aims to compare the algebraic operation skills of mathematics teachers from Generations X and Y and their awareness levels. The content was analyzed in the study that was designed as a case study. The study participants were 118 secondary school mathematics teachers, 53 teachers from Generation X, and 65 teachers from Generation Y. The findings were evaluated under algebraic skills and algebraic awareness topics. The study results revealed that secondary school mathematics teachers from Generation Y had higher mean scores than teachers from Generation X in terms of the difference between the variable and unknown, algebraic operations, quantification, algebra and patterns, quantification, and inequality subjects in the algebra learning field.

Keywords: Algebra learning field; misconception; mathematics education; values in mathematics education; generations X and Y.

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I. Introduction

I.1. Introduce the problem

Developing algebraic thinking early on in life can increase success significantly, contribute to algebra, and eliminate the problems in the transition period to algebra (Gamoran and Hannigan 2000). Algebraic thinking is the ability to focus on the relationship between the numbers (Kieran 2004; Kieran 2018). Algebra has its grammar and syntax with standardized symbols, signs, and rules (Drijvers et al. 2011). Algebraic reasoning comprises two fundamental elements: Generalizing or articulating concepts through progressively formal and traditional symbolic systems, and manipulating symbols within a structured symbolic framework using an established syntax (Kaput et al. 2008). It is stated that the concepts of generalization, representation, justification, and reasoning become prominent in the mathematical structure and relations related to algebra. It has also been stated that children can think more algebraically at an early age than it is generally believed (Blanton et al. 2018). Algebra facilitates logical thinking, cognitive and logistical growth, the use of mathematical models for solving algebraic problems, the formulation and visualization of patterns, and the development of algebraic language (Dekker and Dolk 2011; Hendroanto et al. 2018; Murray 2010). Better learning of mathematical concepts depends on understanding the meaning of the letters that represent numbers in equations (Clements and Sarama 2004; Moss and Lamberg 2019). Algebraic expressions and equations act as a model to interpret the data and syllogize. In addition, algebra is the foundation of mathematical reasoning that individuals need their whole life (Blanton et al. 2018).

Teaching algebra varies in different periods, and these periods are now referred to as generations. The idea of a generation pertains to individuals and communities born during a specific era, influenced by the social, cultural, economic, and political events of their time, as well as their prevailing values (Altıntuğ 2012). Classification and definition of the concept of a generation that is the subject of various interdisciplinary research vary culturally (Üstün and Taş 2021). Generation X generally refers to the people born between 1968 and 1979, but sometimes the lower limit can also be 1963, and the upper limit can be 1982. Generation Y refers to the people born between 1980 and 1999. The lower limit for Generation Y is 1978, and the upper limit is 2002. Generation Z refers to the people born between 1997 and 2012 (Tolbize 2008). It is natural that the methods and techniques of teachers from different generations who show different characteristics, differ when the description of the generation concept and the different periods of generations are considered. Evaluating and comparing the thinking styles and conceptions of teachers from

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different generations and their perspectives on the lesson, especially in algebra lessons that require abstract thinking skills, are important. While scientists conducted comparisons across various generations, the scarcity of studies in the field of mathematics education underscores the uniqueness of this research.

I.2. Teaching Algebra

Teachers must consider teaching methods at different levels and adapt to changes in teaching to improve and develop students' algebra learning (Litke 2020). In addition to efficient assessment techniques, educators require impactful teaching approaches and resources to gauge the progress of students in algebra (Genareo et al. 2020). The teachers' decisions while responding to student feedback affect how students learn mathematics (Kassel, 2020). It is stated that mathematical modeling or relational tasks established between numbers can be useful in learning algebra (Kaput et al. 2008). Even though it holds significant importance, algebraic thinking remains a formidable skill in both the realms of education and learning (Capraro and Joffrion 2006; Jupri et al. 2014). Learning mathematics includes learning ways of thinking. However, the artificial separation of arithmetic and algebra prevents strong mathematics interpretation and makes learning algebra difficult. If individuals possess a deep understanding of arithmetic, allowing them to articulate and substantiate its properties in the context of mathematics, they will have acquired essential underpinnings for the study of algebra (Carpenter et al. 2003). One of the reasons for the difficulties in learning algebra is that the individuals' over-generalizing the mathematical rules acquired while learning the subject using their previous knowledge (Barbieri et al. 2019; Stagylidou and Vosniadou 2004).

I.3. Misconceptions in Algebra

Misconceptions are erroneous ideas that individuals have on any phenomenon that is inconsistent with scientific concepts (Kieran 2014). It is possible to define misconception as incorrect applications or generalization of a rule (Drews 2008). Individuals harboring mathematical misconceptions typically possess an inaccurate or partial comprehension of fundamental mathematical concepts (Barbieri et al. 2019). Proper understanding of the concepts is important in understanding the concepts to be learned later. Incomplete or incorrectly learned concepts can affect the following subjects to be learned (Nasution 2019). It is essential to understand the reasons for misconceptions that hinder learning and clarify them (Irawati and Ali 2018). Misconceptions can make developing algebraic problem-solving skills a challenge (Booth and Koedinger 2008). Algebraic misconceptions obstruct the acquisition of crucial concepts for algebra success (Russell et al. 2009).

I.4. Aim and sub-problems

The research aims to compare the algebraic operation skills and awareness levels of secondary school teachers from Generations X and Y. The answers to the following questions were sought in accordance with this aim:

1. How do mathematics teachers from different generations perceive algebra?
2. What is the awareness of mathematics teachers from different generations about algebra?

II. Method

The study was designed using the phenomenological approach, a qualitative research method. Phenomenology focuses on phenomena that we are aware of but do not have a deep or detailed understanding of. Therefore, when considering the aims and objectives of the research, it is clear that the phenomenological design is appropriate for this study. Indeed, in phenomenological research, the primary data collection tool is the interview (Yıldırım and Şimşek 2016). In phenomenological research, data sources are individuals or groups experiencing the phenomenon that is the focus of the research. Interviews are conducted to uncover the experiences associated with the phenomena (Büyüköztürk et al. 2016). Accordingly, in this study, the algebraic abilities and awareness of teachers from different generations were taken and compared by asking their opinions.

II.1. Data collection tools

The data collection instrument utilized in this study consisted of three sections: the first section gathered demographic information about the teachers, the second section measured their algebraic abilities, and the third section assessed their algebraic awareness. The "Algebra Learning Area Questionnaire" employed in the study received ethical approval from the Bingöl University Scientific Research and Publication Ethics Committee under the reference number 92342550/108.01/11276. While developing the form, five academicians who are experts in the field of mathematics education were consulted to ensure its validity. As a result of the interviews, the algebra learning field questionnaire was finalized. The study data collection was conducted online.

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II.2. Participants

The study was conducted with 118 secondary school mathematics teachers in total (52 males, 66 females), 53 teachers from Generation X, and 65 teachers from Generation Y. Teachers' ages were considered as a determining factor of their generations. The participants were included in the study based on the convenience sampling method. The convenience sampling method is adopted as a method that provides convenience in time and effort, in terms of economy and usefulness (Büyüköztürk et al. 2016). The teachers from Generation X were coded as TX, and teachers from Generation Y were coded as TY. Table 1 provides information about the participants.

Table 1

Descriptive statistics about the participants

Generation	Gender	Years of teaching experience	n
Generation X	Female	20-23 Years	7
		24-27 Years	5
		28-30 Years	10
		30+Years	8
	Male	20-23 Years	4
		24-27 Years	9
		28-30 Years	6
		30+Years	4
Total			53
Generation Y	Female	1-4 Years	10
		5-8 Years	9
		9-12 Years	10
		13+ Years	7
	Male	1-4 Years	9
		5-8 Years	10
		9-12 Years	5
		13+ Years	5
Total			65

II.3. Data analysis

The data underwent analysis through the method of content analysis. The data obtained through content analysis should be conceptualized first, and then arranged according to the concepts, and the themes about the data should be identified accordingly (Yıldırım and Şimşek 2016). Content analysis, a method facilitating indirect exploration of human behavior and nature, is characterized as a systematic, replicable approach. It involves summarizing specific words within a text into smaller content categories through coding, following predefined rules. This analytical process is employed to ascertain the presence of particular words or concepts within a single text or a cluster of texts (Büyüköztürk et al. 2016). Initially, the qualitative analysis tool NVivo was utilized, and ongoing analytic memos were crafted to document emerging patterns (Miles and Huberman 1994). During data analysis, repetition of a teacher recounting the same narrative regarding a response or if two individuals mentioned identical accounts were treated as a single case. Additionally, the reliability of the qualitative data was gauged using the formula devised by Miles and Huberman (1994): Consensus Percentage (P) = Consensus / [Consensus + Disagreement]. Three researchers independently analyzed the data, revealing a reliability score of 0.91 among the coders. Instances of disagreement were deliberated upon by the researchers, leading to a recoding process to derive findings. Direct quotations from the interviews are incorporated in the findings section to substantiate the obtained data.

III. Results

III.1. Algebraic skills

In the research, the themes of knowing the difference between the unknown and the equations, operation skills, quantifying verbal expressions, the concept of patterns, identity, minimum and maximum values, and inequality of mathematics teachers from Generations X and Y were discussed. Figure 1 shows the findings.

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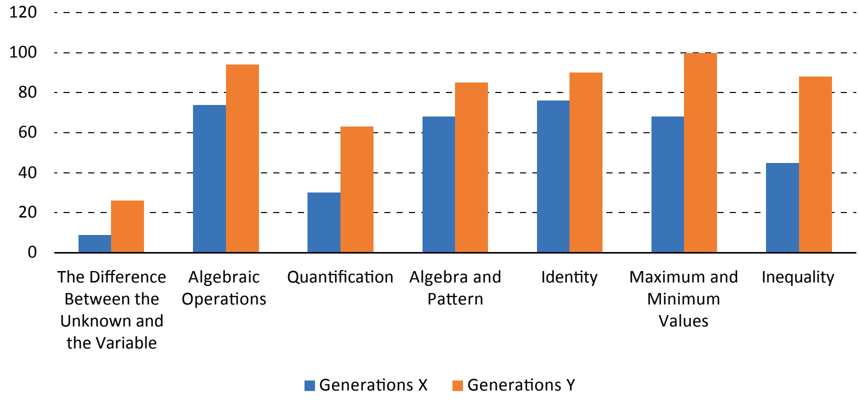

Figure 1

Algebraic skills

III.2. The Difference between the unknown and the variable

The teachers were asked to explain the difference between the unknown and variables and asked to find the unknowns and variables by being provided with several premises. Approximately 9% (f = 5) of the mathematics teachers from Generation X and 26% (f = 17) of mathematics teachers from Generation Y know the difference between unknown values and variable concepts in algebraic expressions. On the other hand, most teachers from both generations (X; 91%, Y; 74%) do not know the difference.

III.3. Algebraic operations

A problem statement with premises that can cause misconceptions was given to the mathematics teachers from Generations X and Y to identify their misconceptions about their algebraic operation skills. Approximately 74% (f = 39) of the mathematics teachers from Generation X and 94% (f = 61) of the mathematics teachers from Generation Y know the operations in algebraic expressions flawlessly. Approximately 17% (f = 9) of the teachers from Generation X, and 6% (f = 4) of the teachers from Generation Y who answered incorrectly, did not realize the division by the coefficient of the unknown in the $(4 + 3x) - (2 + 5x) = 0$ operation, and answered incorrectly. Approximately 9% (f = 5) of the teachers from Generation X stated that the correct answer was not among the options and answered incorrectly. In general, calculation errors in the incorrect answers given by the teachers were observed.

III.4. Quantification

In the study, the following two multiple-choice problems were asked to the mathematics teachers from Generations X and Y to determine their skills of quantifying verbal expressions into algebraic expressions: "A student has a weekly allowance of X TL. She divides the allowance equally per the weekdays during which she goes to school. However, since the last two days of school are the International Day of Persons with Disabilities, she first donates half of her money and puts one-third of her remaining money in her piggy bank. How much is the allowance of the student who saves 40 TL at the end of a week?" and "An individual who wants to measure the area of her rectangular field with a string of y unit length, measures the long side of the field as $5y$ length and the short side as $3y + \frac{y}{2}$ length. How many y^2 units is the area of the field?" Approximately 30% (f = 16) of the teachers from Generation X and 63% (f = 41) of the teachers from Generation Y can quantify verbal expressions into algebraic expressions. However, approximately 8% (f = 4) of the teachers from Generation X and 31% of the teachers from Generation Y (f = 20) did not notice the inverse operations in the problem statement and answered incorrectly. Similarly, approximately 17% (f = 9) of the teachers from Generation X and 13% (f = 7) of the teachers from Generation Y had misconceptions in the inverse operation point by choosing one of the incorrect options. In addition, approximately 17% (f = 9) of the teachers from Generation X made a calculation error and found the answer twice as much, hence, answering incorrectly. When the answers of the teachers who answered incorrectly were evaluated, it was determined that they made an error in the quantification of "However, since the last two days of school are the International Day of Persons with Disabilities, she first donates half of her money and puts one-third of her remaining money in her piggy bank." sentence while forming the equation.

III.5. Algebra and pattern

In the study, the two following questions were asked to the mathematics teachers from Generations X and Y to determine their skills to express the general term of an arithmetic pattern and the general term of the shape pattern algebraically: "Express the general term of the arithmetic pattern of which the first five terms are 5, 12, 19, 26, 33 algebraically." and "What is the algebraic rule of the relationship between the number of octagons and the total number of sides in the shape pattern created by adding the octagon so that it has a common edge with only one of the existing octagons at each step?". Approximately 91% (f = 48) of the teachers from Generation X, and 100% (f = 65) of the teachers from Generation Y have the skill to express the general

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term of a pattern algebraically. However, approximately 9% (f = 5) of the teachers from Generation X tried one or a few steps of the pattern through trial and error and did not think that all steps should be applied for the general term and made a misconception error. Approximately 45% (f = 24) of teachers from Generation X, and 69% (f = 45) of the teachers from Generation Y can express the general term of a shape pattern algebraically. However, approximately 47% (f = 25) of the teachers from Generation X, and 14% (f = 9) of the teachers from Generation Y thought that the 8n-1 distractor was correct based on the number of sides of the octagon, and they misunderstood by not considering the common edges. Similarly, approximately 8% (f = 4) of the teachers from Generation X, and 17% (f = 11) of the teachers from Generation Y had a misconception while expressing the general term of the shape pattern algebraically and stated that the correct answer was not among the options.

III.6. Identity

In the study, the following questions were asked to the mathematics teachers from Generations X and Y to determine their skills to expand squared brackets, and knowing the difference of two squares identity, and quadratic equations: *"Please expand* $(3a-2b)^2$" and "*What is the sum of x integer values in the expression* $x^2 - 64 = 3x - 2(x + 4)$*?"*. Approximately 83% (f = 44) of the mathematics teachers from Generation X, and 92% of the mathematics teachers from Generation Y (f = 60) know expanding squared brackets. However, approximately 8% (f = 4) of teachers from Generation X and 8% of teachers from Generation Y (f = 5) miscalculated the twofold of two terms multiplied in brackets expansion. Similarly, approximately 9% (f = 5) of the teachers from Generation X made a sign error when calculating twice of the two terms in bracket expansion. When the answers given to the problem were examined, it was seen that the teachers who answered incorrectly did not know the proposition "no simplification with zero" and simplified the factors $(x - 8)$ with each other, so they did not consider one root of the equation. When the answers given to the question about the squared bracket expansion were examined, it was seen that some of the teachers who made mistakes did not notice that twofold of the first and second statements in the series should be considered.

III.7. Maximum and minimum values

In the study, the following question was asked to determine the skill of calculating the greatest and smallest value in algebraic expressions of mathematics teachers from Generations X and Y: "*Find the maximum and minimum values of the product x.y for the natural numbers x and y in the expression* $3x + 2y = 18$". Approximately 68% (f = 36) of the mathematics teachers from Generation X, and 100% (f = 65) of the mathematics teachers from Generation Y know how to calculate the maximum and minimum values in algebraic expressions. Misconceptions about the definition of natural numbers were observed when the answers of teachers from Generation X were analyzed.

III.8. Inequality

In the study, the following question was asked to determine the inequality in algebraic expressions skills of mathematics teachers from Generations X and Y: "If x is an integer, and $x + 2x + 3x + \ldots + 10x > 55$, then what is x^2 at least?". Approximately 45% (f = 24) of the mathematics teachers from Generation X and 88% (f = 57) of the mathematics teachers from Generation Y know the concept of inequality. When the incorrect answers were analyzed, it was seen that teachers used x instead of x^2 or overlooked the fact that x being an integer.

III.9. Algebraic awareness

To investigate teachers' algebraic awareness, the study compared responses to questions about variable and unknown concepts, misconceptions and values between Generation X and Generation Y. Results detailing common and divergent responses are categorized into clusters. Figure 2 shows the themes identified regarding the awareness of mathematics teachers from different generations regarding variable and unknown concepts, misconceptions and values relevant to algebra education.

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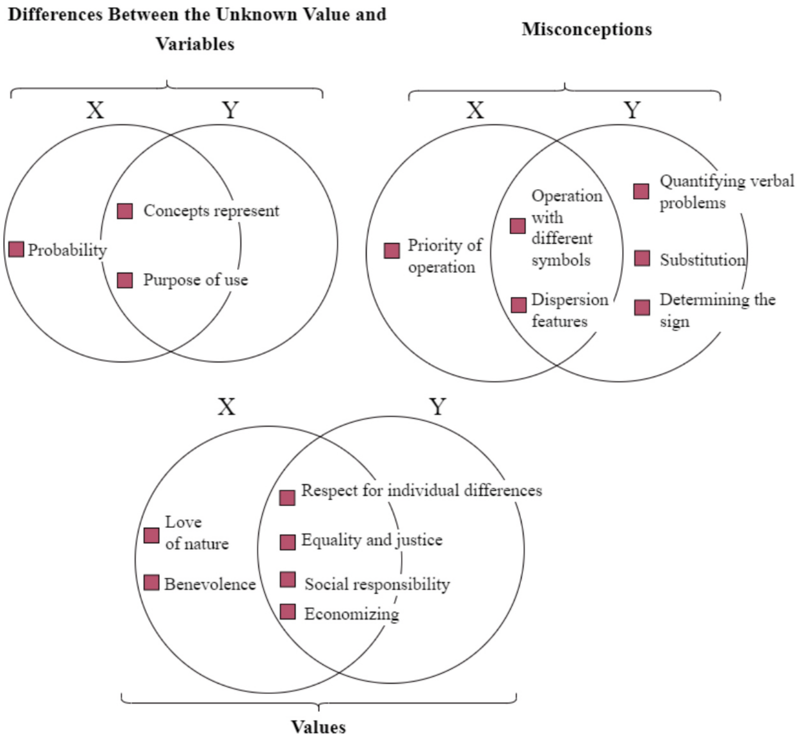

Figure 2

Awareness in algebra education

III.10. Awareness about the differences between the unknown value and variables

While explaining the differences between the unknown value and variables, teachers from Generation Y highlighted the value and purpose of use that these concepts represent, and teachers from Generation X also explained the concept of probability. Sample expressions of teachers who have an awareness of this issue are given below:

TX2: "Variable: A symbol or a cardinality that represents any element of a set of at least two elements. The unknown value corresponds to the cardinality concept defined in mathematics but whose value is unknown."

TX16: "The variables are used for identity in equations with unknown values".

TX24: "While the concept of variable symbolizes the possible potential outcomes in an algebraic expression, the concept of unknown variables symbolizes that the probability is 1."

TY22: "Unknown value is an unchanging letter expression with only one numeric value. Variables are letter expressions whose numerical value can change."

TY25: "In an equation, x is the unknown value. However, x is a variable in identity or algebraic expressions."

III.11. Awareness about misconceptions

Regarding the difficulties encountered in algebra teaching and the misconceptions observed in students, teachers from Generation X stated that the priority of operation, operation with different symbols, and dispersion features became prominent. Teachers from Generation Y expressed the problems they encountered as difficulties in processing with different symbols, processing priority, assigning the equality to the opposite side of the equation, determining the sign, and quantifying verbal problems. Sample expressions are given below:

TX4: "Students usually make mistakes while expanding the brackets. For example, such as $-(2x + 3) = -x + 3$."

TY52: "When distributing algebraic expressions into brackets in multiplication, multiplying only by one element, and forgetting the other one: $2.(3x + 4) = 6x + 4$."

TY56: "Carrying out the operation as if the exponents are not going to be distributed, like in the following example: $2.(3x + 5) = 6x + 5$."

TX32: "Generally, students add different values, like $2a + 2b = 4ab$."

TY36: "For example, the students can calculate as $2a + 3b = 5ab$."

TY12: "Multiplying different terms: $2y.3x = 6x$ *or* $6y$."

TX64: "There are misconceptions about operations with brackets."

TX3: "Difficulties about solving equations due to sequence or priority."

TY1: "For example, if a student cannot calculate the values in brackets, then she has misconceptions about the order of operations."

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TY88: "They have misconceptions while quantifying verbal statements. Let's say that "*f*" denotes females and "*m*" denotes males in the statement that males are 3 times more than females in a class. They can have misconceptions due to misplacing variables like $m=3f$."

TY112: "Students have misconceptions while assigning values to the other side of the equations by changing sign of the terms."

TY20: "Students think that 5 in the statement $5x$ will be assigned to the other side of the equation as a positive value, this is a misconception about the coefficients in algebraic statements."

TX48: "Trying to solve equations without knowing the order of operations."

TX8: "Making an error in the direction of the inequality symbol when multiplying or dividing both sides of the inequality in negatively signed coefficients."

TY20: "The student does not use brackets and creates a misconception when multiplying the excess."

III.12. Awareness about values to be taught

Teachers from Generation X stated that they could teach concepts about equality and justice, social responsibility, respect for individual differences, economizing, benevolence, and love of nature to students, while teachers from Generation Y stated that they could teach equality and justice, social responsibility, respect for individual differences and economizing. Sample statements of teachers from Generations X and Y are given below:

TX2: "Equality and Justice: We have x number of walnuts. We divide x number of walnuts to 5 siblings equally as x divided by 5. In this way, some values can be taught to our students in the lesson."

TX84: "Equality as a requirement of justice and social principles is the first value that comes to my mind."

TY14: "The greengrocer being honest and fair while weighing fruit emphasizes the value of equality in the equations."

TY88: "I associate the concepts of justice, social sensitivity, and responsibility with the concept of equality in the equations."

TX64: "For example, social responsibility values are taught in a problem statement that expresses the amount of blood donated as liters on World Health Day."

TX44: "Different values teach about respecting individual differences."

TY22: "Especially in questions about sharing, we can emphasize social sensitivity and responsibility by emphasizing that they should protect equality, justice, and sensitivity towards their shares against people."

TY72: "A ratio of going out and using masks teaches values of social responsibility."

TY100: "Expressing the amount of water used as x can teach the importance of saving water to students."

TY76: "Simplification can teach about economizing and getting rid of excess materials."

TX24: "The problem statement asking the students to write down the equation showing the amount of bread waste per day in the phrase "At least 5 million bread are wasted per day" can teach about economizing."

TY10: "If there are positive and negative values in an equation, the change in the direction of the inequality in the equation can be explained by the changing the responses given in the current situation due to changing responsibilities in social life."

TY88: "In some optional shopping questions, for example, a mobile operator's monthly tariff options, fixed fee and which option would be more economical in per-minute pricing can teach about economizing to the students."

TX15: "By saying that the pattern has a certain rule and all terms can be found according to these rules, obedience to the rules and social responsibility values can be taught."

TY72: "In a pattern of numbers whose first term is one, the 15th term will be the definitive result for every student. This can teach the value of equality."

TX84: "The value of benevolence can be taught by setting up a problem sentence with the related concepts."

TX24: "A simple subject of the equation can be taught together with the value of economizing."

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TX3: "Examples such as the sunflowers can be given as examples so that the students explore the mathematics in nature. Therefore, love of nature can be taught."

IV. Discussion

Upon evaluating teachers' responses regarding their opinions on the distinction between the unknown value and the variable, the research determined that Generation Y educators exhibited greater proficiency, while their Generation X counterparts demonstrated a lower awareness of this distinction. While teachers from Generation Y highlighted the values and purpose of using the unknown values and variable concepts in explaining the difference, teachers from Generation X also explained the probability concept. The increasing importance of the explanations of mathematical terms and the more aware use of these terms are the reasons for these results. The unknown value and variable terms are employed to explain different uses of characters in algebraic expressions and equations. Generally, the unknown value term is used for explaining a certain amount, and even though a person does not know the value yet, it refers to a numerical value that can be identified using the information provided. A variable is not definite but indefinite. For example, in an equation dependent on x and y variables, equality is provided for the infinite values of x and y. On the other hand, together with the y value found for a value to be given x, the pair (x,y) represents a part of the general set (Ely and Adams 2012). In addition, learning different meanings of characters and variables was stated as important (Blanton 2008; Blanton et al. 2017). In algebra, understanding how algebraic equations are solved and understanding which situations algebra represents are necessary (Moss and Lamberg 2019). Using symbols correctly and establishing a correct relationship between the symbols are important while learning algebra (Malara and Navarra 2012).

When the misconceptions of mathematics teachers from Generations X and Y were researched, it was concluded that teachers from Generation Y had fewer misconceptions than teachers from Generation X. Teachers from Generation X had more misconceptions while dividing by the coefficient of the unknown while doing mathematical operations compared to the teachers from Generation Y. In addition, teachers from both generations made calculation errors. Mistakes made in the process of associating and retrieving cognitive information could be the reasons for calculation errors. As a matter of fact, in cognitive information processing, individuals focus on associating the information with data in memory, storing new information in memory, and retrieving information when needed (Schunk 2008). Mathematics teachers from Generation Y were more successful in quantifying verbal statements into algebraic expressions than mathematics teachers from Generation X. However, mathematics teachers from Generation X had fewer misconceptions than mathematics teachers from Generation Y in terms of inverse operations. The fact that a semantic relationship is established through symbols and the development of awareness through mental processes such as inverse processing are the reasons for this result. Knuth et al. (2006) stated that learning the meanings of symbols and variables is necessary for algebraic competency.

Teachers from Generation Y were more successful in expressing the general term of an arithmetic pattern and the general term of a shape pattern algebraically. Teachers from Generation X try one or several steps of a pattern via trial and error and have misconceptions by not thinking that all the steps should be carried out for finding the general term. Teachers from Generation X had more misconceptions than teachers from Generation Y while considering the common sides of a shape while expressing the general term of the shape pattern algebraically. Patterns are important for algebra and require advanced cognitive skills. As a matter of fact, difficulties experienced while learning algebra can also make it hard to learn advanced algebra subjects (Herriott and Dunbar 2009).

It was concluded that teachers from Generations X and Y had similar skills of expanding squared brackets, knowing the difference of two squares identity and quadratic equations, and teachers from Generation Y experienced fewer misconceptions. Among the teachers of both generations, some teachers had misconceptions in expressing the twofold of the two terms multiplied. When the solutions were analyzed, it was seen that teachers who answered incorrectly did not know the proposition of "no simplification with zero", and overlooked one root of the equation. In addition, teachers also had misconceptions about negativity. Correctly understanding the meaning of the equals sign is necessary for learning algebraic equations and solving questions (Carpenter et al. 2003). It has been mentioned that individuals who misinterpret or have an incomplete understanding of the negative sign in algebra are prone to employing incorrect strategies when solving algebraic equations (Booth and Koedinger 2008). While teachers from Generation Y calculate the maximum and minimum values in algebraic expressions without any misconceptions, most teachers from Generation X were also successful. When the incorrect answers given by teachers from Generation X were analyzed, misconceptions about the definition of natural numbers were identified. While teachers find the maximum and minimum values, they cognitively force their mental schemas and may have misconceptions. As a matter of fact, misconceptions are expressed as commonly accepted situations among different generations and cause mathematics to be considered a challenging course (Blanton et al. 2018).

Teachers from Generation Y were more successful in their skills of inequality in algebraic expressions than teachers from Generation X, and it was observed that teachers overlooked forces of x or x being an integer. Misconceptions need to be corrected to know the inequality concept in the best way possible. As a matter of fact, learning equals sign completely is stated as important for developing algebraic problem-solving skills (Knuth et al. 2006). In addition, one of the biggest difficulties faced while transitioning to algebra from arithmetic's is understanding how a mathematical equation is formed (Lee et al. 2018). Teachers from Generation X highlighted the order of operations, solving operations using different symbols and distributive property in terms of difficulties faced during algebra teaching and misconceptions of teachers. Teachers from Generation Y deemed the problems they face in solving operations using different symbols, order of operations, assigning the values to the other side of the equation, determining signs, and quantifying verbal problems as important. Lack of algebra can cause problems and misconceptions (Apsari et al. 2020) of calculation (Müller et al. 2014), mathematical proof (Güler, 2016), and solving problems (Ferryansyah et al. 2018).

Teachers from Generation X stated that equality and justice, social responsibility, respect for individual differences, economizing, benevolence, and love of nature could be taught to students while teaching algebra and expressed that these values are important. Teachers from Generation Y stated that values of equality and justice, social responsibility, respect for individual differences, and economizing could be taught via algebra education and stated verbally that these values should be highlighted. The importance given to the concept of value in the recent education curricula, and highlighting mathematical values in national and international exams are the reasons for these results. Another reason for these results can be the fact that values are important for both generations, they adopt these values and pass them on to Generation Z while teaching. As a matter of fact, in the literature, Generation X is generally described as a generation that has continuity in learning, has strong technical skills, attaches importance to individuality and values, and is result-oriented, and while Generation Y shares many characteristics of Generation X, they are also defined as a generation that values teamwork, optimistic, can take flexible decisions and individuals with high sense of self-esteem (Bova and Kroth 2001; Crampton and Hodge 2006; Glass 2007; Martin 2005; Tolbize 2008; Üstün and Taş 2021).

V. Limitations and future directions

The applications were carried out using online forms due to the pandemic, but teachers were contacted individually to avoid this limitation. While preparing the questionnaire, it was stipulated that each question should be solved and open-ended questions should be answered, and data loss was prevented in this way. The teachers who participated in the study agreed to participate in the process voluntarily, and declared their choices, and ethical permission was obtained. Teachers were asked to solve multiple-choice problems and write down their solutions. Hence, the data were analyzed in detail. The results obtained show that teachers lack in certain areas in terms of similarities, differences and definitions of important concepts such as unknown values and variables. To overcome these failings, studies on teacher training programs can be carried out. Academic projects can be prepared to develop the skills of in-service teachers. In the study, important results were obtained regarding the misconceptions of the teachers from Generations X and Y. The differences show that there is a need for new studies on this subject. To reduce the current generation differences, lifelong learning programs for teachers can be developed. These programs can increase the teachers' awareness of the values of mathematics education. Based on the research findings, policymakers can develop professional development programs tailored to the needs of mathematics teachers and prospective teachers. These programs can equip teachers with the knowledge and skills they need to effectively teach algebra. Additionally, education stakeholders can facilitate the creation of collaborative learning environments adapted to the characteristics and needs of Generation X and Y teachers.

VI. Conclusions

This study reveals discrepancies in the knowledge and skills of mathematics teachers across generations. These variations could potentially impact the teaching of algebra. The findings emphasize the necessity of reviewing teacher education and policies. The need to develop new strategies tailored to the identified challenges, problems, and needs has been frequently emphasized (Adamu 2021; Özmutlu 2022). It is expected to encounter differences among teachers with diverse backgrounds and experiences. The crucial aspect lies in identifying and acknowledging these differences, and utilizing them to promote the development of effective learning and teaching methods.

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